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Power efficient algorithms for wireless charging under phase shift in the vector model

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***Abstract*—Recent technological advances in the domain of Wireless Power Transfer (WPT) have enabled the employment of previously unrealistic methods for power management in wireless systems. At the same time, some of the classical scalar models have proved incapable of capturing the multi-dimensional aspects of WPT that are similar to the superposition of wave functions. In this work, we consider the *vector model* which is by now a widely accepted model for WPT and its validity has been confirmed experimentally in the literature.**

Under the vector model, we study the problem of power maximization in a wireless network consisting of wireless chargers. We take the state of the art one step further by assuming that chargers can use phase-shifting to adjust their output in order to improve the total power provided by the network of chargers at selected points in the network area. Even though the technology for phase-shifting already exists, researchers have only recently tried to study it from an algorithmic perspective and algorithmic solutions are nearly inexistent. In this paper, we provide a rigorous formulation for the problem of power maximization as a semi-definite program with rank constraints and we present efficient centralized and distributed solutions, and also heuristics where only local information is available.

I. Introduction

Wireless power transfer is gradually becoming a mature technology that is used in networks of power-thirsty wireless devices, to deliver energy and keep the network functional. It is evident, that the ability of providing stable voltage output and continuous power, without the need for maintenance procedures, has made it an excellent choice for various IoT applications which prohibit the use of a cable. To give an idea of the growing potential of this particular technology, we mention that the Wireless Power Consortium currently counts 659 members [1] including several of the world's largest companies in the technology field and beyond, while the wireless power transmission technology market is estimated to grow at a Compound Annual Growth Rate (CAGR) of 23.15% between 2017 and 2022.

In general, a WPT network consists of a set of wireless chargers and receivers which are deployed in an area of interest. A wireless charger generates a time-varying electromagnetic field, and transmits power across free space to potential receiver devices. There is a large amount of relevant publications addressing problems regarding WPT, but only a small subset of those study WPT from a purely rigorous and algorithmic perspective. In the past, this was mainly due to the fact that there were no realistic enough models that could on one hand capture the characteristics of WPT and on the other hand be simple enough to allow non-trivial analysis. Recently, the research landscape has changed due to the emergence of such a new model capable of describing wireless power transfer technology more accurately, by taking into account fundamental properties of electromagnetic waves and their precise interactions.

The so-called *vector model* captures constructive and destructive electromagnetic phenomena which are generated by the electromagnetic waves emitted by the chargers, making it appropriate for far-field applications via electromagnetic radiation. Problems such as power maximization can now be resolved more efficiently, achieving significant improvements over other solutions that rely on less descriptive models.

Our Contribution. Our contribution in this paper is the following:

- In Section III, building on the gradually expanding related literature ([15]–[21]), we extend the vector model to accommodate for chargers that are initiated at different times, inducing a *phase shift* on their generated electric fields.
- Assuming the vector model with phase shifts, we present in Section IV the `PowerShift` problem of finding a phase shift configuration that maximizes the total power from chargers to nodes in the network. We demonstrate in the evaluation section that phase shifting can significantly increase the WPT network QoS in terms of cumulative power received.
- We formulate this problem as a semi-positive definite maximization program with rank constraints (see Section V). Even though there are no standard tools for solving this formulation of the problem (because of the rank constraint), we prove that it can be used to efficiently find

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an upper bound to the optimal solution of PowerShift.
- In Section VI, we present a randomized approximation solution based on relaxation and probabilistic rounding. Our probabilistic rounding approach is novel and heavily relies on theoretical results on multivariate normal random variables.
- Furthermore, in Section VII, we present a distributed algorithm (DASA) for computing an approximation to the optimal solution to PowerShift and we show that all required computation can be performed locally by every charger, assuming nodes can send information regarding the vector of the electric field created at their location.
- We conclude our algorithmic study by presenting in Section VII another distributed algorithm (DAH) that only allows chargers to use phase shifts belonging to (small) predefined sets of values.
- Finally, in Section VIII, we evaluate the performance of our algorithms and heuristics by extensive simulations.

II. Related work

In recent years there has been a radical improvement of wireless charging technology. More and more companies are now framing Powercast [3], one of the first to launch this venture. Texas Instruments [4] and Analog devices [5] have also announced circuits that mainly intended for Industrial, Scientific and Medical (ISM) wireless power applications in far field via RF waves.

We note that, the theory behind WPT is not new; similar phenomena have been studied in wireless radio communication settings (see [7]), where a wireless sensor network with resource-limited devices exposed to interference can select an appropriate mitigation strategy. Popular wireless charging technologies such as electromagnetic induction have also been applied to communications ([6]). In [8] a hybrid system is presented that combines both communication and harvesting. We therefore can say that there is a close correlation between wireless charging and wireless communications technologies since both share characteristics pertaining to waves in physics.

An equally important problem in this field, is the placement of chargers in a WPT network, where works like [10] and [13] are designed to improve charging quality. Similarly, in [12], a study is conducted concerning the trade-off between maximum energy provided in the network and fair distribution of energy, with respect to the chargers' location. In addition, the authors in [11], besides the placement part, also study the corresponding power allocation to maximize the charging quality, subject to a power budget. In [14] an analysis is presented regarding the placement of the chargers in such a way that radiation is kept below a threshold, while in [9], the authors aim to maximize the overall expected charging utility for all points by determining the positions and orientations of chargers.

Finally, we note some works that seem to be closer to ours, focusing on more efficient WPT networks and sharing our common interest for applying more realistic models. Such models (including the vector model studied in this paper) capture fundamental properties of the superposition of energy fields and allow a more precise calculation of the power at the terminals of a receiver antenna, than other one-dimensional models that have been used in the past and ignore *superadditive* and *cancellation* effects. [15] provides an experimental study regarding charging, packet loss due to interference, and suitable ranges for charging and data communication of the energy transmitters. It also explores how the placement and relative distances of multiple ETs affect the charging process. The model was also exploited in [16], to solve the problem of maximizing power in a WPT network by regulating the chargers' operation level, while recently in [17] the authors study the Minimum Radiation Path Problem with respect to vector model. In [18] an experimental evaluation has been conducted, demonstrating the advance of the vector model against scalar based approaches. In [19], the corresponding analysis shows that the proposed deployment methods, by exploiting the physical characteristics of wireless recharging, can greatly reduce the number of the reader nodes, compared with other assuming traditional coverage models.

III. The vector model with phase shift

In [2], the authors considered a model that describes the distributions of total available and harvested power over an entire WPT network and the corresponding constructive and destructive interference of the transmitted energy signals. Their model captures fundamental wave-like properties of electromagnetic fields and its basic idea is that it uses 2-dimensional vectors as a mathematical abstraction for electromagnetic fields, hence the name *vector charging model* (or just *vector model*). The vector model (which was studied algorithmically for the first time in [16] by authors of the current paper) has been experimentally shown to provide a more accurate abstraction than the one-dimensional model suggested by Friis' formula for the power received by one antenna under idealized conditions given another antenna some distance away (see [18]). However, this larger expressibility comes together with higher complexity in comparison to other models.

Building on the gradually expanding related literature ([15]–[21]), the vector model can be extended to accommodate for chargers that are initiated at different times, inducing a *phase shift* on their generated electric fields. In particular, the *electric field* created by an energy transmitter (charger) C, with phase shift $\phi_C \in [0, 2\pi)$, at a receiver R (or point in space) at distance $d = \text{dist}(C, R)$ is a 2-dimensional vector given by

$$\mathbf{E}_{\phi_C}(C, R) \stackrel{\text{def}}{=} \beta \cdot \frac{1}{d} \cdot e^{-j\frac{2\pi}{\lambda}d+\phi_C} \quad (1)$$

where λ depends on the frequency at which the transmitter operates, β is a constant that depends on the hardware of the transmitter and the environment.

Consequently, the *total electric field* at a receiver R from a set of m chargers $\mathcal{C} = \{C_1, C_2, \ldots, C_m\}$ is merely the

superposition (i.e. the vector sum) of their individual electric fields, namely

$$\mathbf{E}_{\mathbf{\Phi}}(\mathcal{C}, R) \stackrel{\text{def}}{=} \sum_{C \in \mathcal{C}} \mathbf{E}_{\phi_C}(C, R), \tag{2}$$

where $\mathbf{\Phi} = (\phi_{C_1}, \phi_{C_2}, \ldots, \phi_{C_m})$ is the m-tuple of phase shifts (where ϕ_{C_i} is the phase shift for charger C_i, $1 \leq i \leq m$). We refer to $\mathbf{\Phi}$ as the *phase shift configuration* of the chargers.

Furthermore, since the power is analogous to the square of the length of the electric field, the total available *power* at the receiver R from the set $\mathcal{C}$ of chargers operating at full capacity in phase shift configuration $\mathbf{\Phi}$ is given by

$$P_{\mathbf{\Phi}}(\mathcal{C}, R) = \gamma \cdot \|\mathbf{E}_{\mathbf{\Phi}}(\mathcal{C}, R)\|^2, \tag{3}$$

where $\|\cdot\|$ denotes the length (2-norm) of a vector. The constant γ depends on the hardware of the transmitter, the hardware of the receiver and the RF-to-DC conversion efficiency. Finally, we assume that power is additive, namely the *cumulative power* provided by a set $\mathcal{C}$ of chargers in phase shift configuration $\mathbf{\Phi}$ to a set of n receivers $\mathcal{R} = \{R_1, R_2, \ldots, R_n\}$ is given by

$$P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R}) = \sum_{R \in \mathcal{R}} P_{\mathbf{\Phi}}(\mathcal{C}, R). \tag{4}$$

IV. Problem Definition

Consider a system consisting of a family $\mathcal{C} = \{C_1, C_2, \ldots, C_m\}$ of m identical wireless chargers and a family $\mathcal{R} = \{R_1, R_2, \ldots, R_n\}$ of n identical wireless receivers (nodes). Observe that, by the definition of the vector model with phase shift described in equations (1) to (4), different phase shifts for each charger can give different values for the cumulative power $P(\mathcal{C}, \mathcal{R})$. In fact, the change in the cumulative power as a result from shifting from a phase shift configuration $\mathbf{\Phi}$ to another configuration $\mathbf{\Phi}'$ can be significant, as shown in the evaluation section Figures 2, 3.

Based on the above observation, we assume here that phase shifts can be adjusted for each charger so that specific quality of service is provided. In this paper, in particular, we study the following computational problem:

Definition IV.1 (Power maximization with power shifts (`PowerShift`))**.** Given a system consisting of a set of chargers $\mathcal{C}$ and a set of receivers $\mathcal{R}$, find a phase shift configuration that maximizes the total cumulative power from $\mathcal{C}$ to $\mathcal{R}$. That is, find $\mathbf{\Phi}^*$ such that

$$\mathbf{\Phi}^* \in \arg \max_{\mathbf{\Phi} \in [0, 2\pi]^{\mathcal{C}}} P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R}). \tag{5}$$

V. PSD program formulation for PowerShift

In this section we prove Theorem 1, namely that there exists a PSD program formulation with rank constraints whose optimal value is equal to the optimal value of `PowerShift`. To this end, let $\mathbf{P}_\phi = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$ be the 2-dimensional counterclockwise rotation through an angle ϕ. In particular, $\mathbf{P}_\phi = \cos\phi \cdot \mathbf{P}_0 + \sin\phi \cdot \mathbf{P}_{\pi/2}$ and also, using trigonometric equations on equation (1), for any wireless charger C with phase shift ϕ_C the electric field created by C at a receiver R at distance $d = \text{dist}(C, R)$, can be written as follows:

$$\begin{aligned} \mathbf{E}_{\phi_C}(C, R) &= \mathbf{P}_{\phi_C} \cdot \mathbf{E}_0(C, R) \\ &= \cos\phi_C \cdot \mathbf{E}_0(C, R) + \sin\phi_C \cdot \mathbf{P}_{\pi/2} \cdot \mathbf{E}_0(C, R) \\ &= \cos\phi_C \cdot \beta \frac{1}{d} \begin{bmatrix} \cos\left(\frac{2\pi}{\lambda} d\right) \\ -\sin\left(\frac{2\pi}{\lambda} d\right) \end{bmatrix} + \sin\phi_C \cdot \beta \frac{1}{d} \begin{bmatrix} \sin\left(\frac{2\pi}{\lambda} d\right) \\ \cos\left(\frac{2\pi}{\lambda} d\right). \end{bmatrix} \end{aligned}$$

Therefore, by equation (4) and (6), the cumulative power from $\mathcal{C}$ with phase shift configuration $\mathbf{\Phi}$ to the set of nodes $\mathcal{R}$ is given by

$$P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R}) = \sum_{R \in \mathcal{R}} \left\| \sum_{C \in \mathcal{C}} \cos\phi_C \cdot \mathbf{E}_0(C, R) + \sin\phi_C \cdot \mathbf{P}_{\pi/2} \cdot \mathbf{E}_0(C, R) \right\|^2 \tag{6}$$

where $\|\cdot\|$ denotes the Euclidean norm.

Using now the well known fact that, for any real numbers $a, b \in \mathbb{R}$, $a^2 + b^2 = 1$ if and only if there is an angle $\phi \in [0, 2\pi)$ such that $\cos(\phi) = a$ and $\sin(\phi) = b$, we derive the following optimization program for the maximization of $P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R})$:

$$\begin{aligned} \text{maximize} \quad & \sum_{R \in \mathcal{R}} \left\| \sum_{C \in \mathcal{C}} a_C \cdot \mathbf{E}_0(C, R) + b_C \cdot \mathbf{P}_{\pi/2} \cdot \mathbf{E}_0(C, R) \right\|^2 \\ \text{subject to:} \quad & a_C^2 + b_C^2 = 1, \text{for all } C \in \mathcal{C}, \end{aligned}$$

where we replaced $\cos\phi_C$ and $\sin\phi_C$ by a_C and b_C respectively in equation (6).

Notice that the above maximization program can be written in matrix form as follows: We assume an arbitrary ordering of the chargers in $\mathcal{C}$, say $C_1, C_2, \ldots, C_m$. Let $\mathbf{X}$ be a $2m$-dimensional column vector consisting of the variables a_C and b_C, for all $C \in \mathcal{C}$. In particular, assume without loss of generality that the first m entries of $\mathbf{X}$ correspond to the a-values (cosine) $a_{C_1}, a_{C_2}, \ldots, a_{C_m}$ and the last m correspond to the b-values (sine) $b_{C_1}, b_{C_2}, \ldots, b_{C_m}$. Let $\mathbf{Z}$ be the $2m \times 2m$ *symmetric* matrix defined as follows:

$$\mathbf{Z}_{i,j} = \begin{cases} \sum_{R \in \mathcal{R}} \mathbf{E}_0(C_i, R)^T \mathbf{E}_0(C_j, R), & \text{if } i, j \leq m \\ \sum_{R \in \mathcal{R}} \mathbf{E}_0(C_i, R)^T \mathbf{P}_{\pi/2} \mathbf{E}_0(C_j, R), & \text{if } i \leq m, j > m \\ \sum_{R \in \mathcal{R}} \mathbf{E}_0(C_j, R)^T \mathbf{P}_{\pi/2}^T \mathbf{E}_0(C_i, R), & \text{if } i > m, j \leq m \\ \sum_{R \in \mathcal{R}} \mathbf{E}_0(C_j, R)^T \mathbf{E}_0(C_i, R), & \text{if } i > m, j > m \end{cases}$$

where $(\cdot)^T$ denotes transposition. $\mathbf{Z}$ will be used to write the objective function in matrix form. Finally, to write the constraints in matrix form, for every $i = 1, 2, \ldots, m$, we define a $2m \times 2m$ matrix $\mathbf{W}_{C_i}$ that is 0 everywhere except in entries $\mathbf{W}_{C_i}[i, i]$ and $\mathbf{W}_{C_i}[m + i, m + i]$ where it is equal to 1. We now observe that our maximization problem can be written as follows:

$$\begin{aligned} \text{maximize} \quad & \mathbf{Z} \bullet (\mathbf{X} \cdot \mathbf{X}^T) \\ \text{subject to:} \quad & \mathbf{W}_{C_i} \bullet (\mathbf{X} \cdot \mathbf{X}^T) = 1, \quad \text{for all } i = 1, 2, \ldots, m \end{aligned} \tag{7}$$

where $\bullet$ denotes the matrix operation $\mathbf{A} \bullet \mathbf{B} = \sum_{i,j} \mathbf{A}_{i,j} \cdot \mathbf{B}_{i,j}$. Noting that $\mathbf{X} \cdot \mathbf{X}^T$ is a rank 1 Positive Semi-Definite (PSD) matrix, we get the following equivalent PSD program with rank constraints:

$$\begin{aligned} \text{maximize} \quad & \mathbf{Z} \bullet \mathbf{Y} \\ \text{subject to:} \quad & \\ & \mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, \quad \text{for all } i = 1, 2, \ldots, m \\ & \mathbf{Y} \succeq \mathbf{0}, \\ & \text{rank}(\mathbf{Y}) = 1, \end{aligned} \tag{8}$$

where $\mathbf{Y} \succeq \mathbf{0}$ means that $\mathbf{Y}$ is positive semidefinite. We have thus proved the following:

Theorem 1. The optimal solution to `PowerShift` is equal to the optimal solution of the maximization PSD program with rank constraints given in (8).

Notice that, because of the rank 1 constraint, the SDP program (8) cannot be solved using standard tools. Furthermore, by dropping the rank constraint, we get an optimal value that is larger or equal to the solution to `PowerShift`. The difference is that, without the requirement rank$(\mathbf{Y}) = 1$, an optimal solution to the above PSD program can be found in polynomial time (see [22]). Therefore, we have the following:

Corollary 1. There is a polynomial time algorithm that computes an upper bound to the optimal solution of `PowerShift`.

We use the above algorithm described in the Corollary to compare our algorithmic solutions.

VI. ApproxPowerShift: An algorithm via relaxation and probabilistic rounding

As mentioned in the previous section, the PSD program (8) asks for a rank 1 positive semidefinite matrix and so we cannot solve it using standard tools. Instead, we can solve efficiently a *relaxed* version of (8) that we get by dropping the requirement for low rank solutions, namely

$$\begin{aligned} \text{maximize} \quad & \mathbf{Z} \bullet \mathbf{Y} \\ \text{subject to:} \quad & \\ & \mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, \quad \text{for all } i = 1, 2, \ldots, m \\ & \mathbf{Y} \succeq \mathbf{0}. \end{aligned} \tag{9}$$

Unfortunately, an optimal solution to the PSD program (9) may (and will) have full rank, whereas we are looking for a rank 1 solution. The standard way to get the best 1-rank approximation of $\mathbf{Y}$ would be to take the 1-rank matrix corresponding to the largest singular value. However, by doing so there is no guarantee that the constraints $\mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, i = 1, 2, \ldots, m$ of the PSD program (9) remain true (actually it most certainly is not). Therefore, we will instead use a novel probabilistic rounding approach which we describe below, which leads to a polynomial-time probabilistic approximation algorithm.

In particular, let $\mathbf{Y_{opt}}$ be an optimal solution to the PSD program (9) - such a solution can be computed efficiently in polynomial time. We will assume without loss of generality that $\mathbf{Y_{opt}}$ is symmetric. [1] Let $\mathbf{\Psi} \sim \mathcal{N}(\mathbf{0}, \mathbf{Y_{opt}})$, i.e. $\mathbf{\Psi}$ is a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{Y_{opt}}$. By properties of the multivariate normal distribution [23], we have that $\mathbb{E}[\mathbf{Z} \bullet (\mathbf{\Psi} \cdot \mathbf{\Psi}^T)] = \mathbf{Z} \bullet \mathbf{Y_{opt}}$. In particular, $\mathbf{\Psi} \cdot \mathbf{\Psi}^T$ is a rank-1 matrix that achieves on expectation the optimal value of the (full rank) PSD program (9), at the cost of not satisfying (with probability 1) the constraints $\mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, i = 1, 2, \ldots, m$ of the PSD program (9). In particular, we say that $\mathbf{\Psi} \cdot \mathbf{\Psi}^T$ is an *unbiased estimator for* $\mathbf{Y_{opt}}$. Furthermore, recalling notation from equation (7), we have that $\mathbb{E}[\mathbf{Z} \bullet (\mathbf{\Psi} \cdot \mathbf{\Psi}^T)]$ equals

$$\mathbb{E}\left[\sum_{R \in \mathcal{R}} \left\| \sum_{C_i \in \mathcal{C}} \mathbf{\Psi}_{a_{C_i}} \cdot \mathbf{E}_0(C_i, R) + \mathbf{\Psi}_{b_{C_i}} \cdot \mathbf{P}_{\pi/2} \cdot \mathbf{E}_0(C_i, R) \right\|^2\right]. \tag{10}$$

Notice that we use the notation $\mathbf{\Psi}_{a_{C_i}}$ (resp. $\mathbf{\Psi}_{b_{C_i}}$) to denote the entry of $\mathbf{\Psi}$ that corresponds to a_{C_i} (resp. b_{C_i}).

We now enforce that the constraints $\mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, i = 1, 2, \ldots, m$ of the PSD program (9) are satisfied, by normalizing each pair $(\mathbf{\Psi}_{a_{C_i}}, \mathbf{\Psi}_{b_{C_i}})$, for each $i = 1, 2, \ldots, m$, so that their sum of squares equals 1. In particular, we multiply both $\mathbf{\Psi}_{a_{C_i}}$ and $\mathbf{\Psi}_{b_{C_i}}$ by $\frac{1}{l_{C_i}} \stackrel{\text{def}}{=} \frac{1}{\sqrt{\mathbf{\Psi}^2_{a_{C_i}} + \mathbf{\Psi}^2_{b_{C_i}}}}$, for each $i = 1, 2, \ldots, m$, thus getting the new pairs of variables $(\mathbf{\Psi}'_{a_{C_i}}, \mathbf{\Psi}'_{b_{C_i}}) = \left(\frac{\mathbf{\Psi}_{a_{C_i}}}{l_{C_i}}, \frac{\mathbf{\Psi}_{b_{C_i}}}{l_{C_i}}\right)$, for $i = 1, 2, \ldots, m$. Substituting the new variables into (10), we see that the normalization changes the expectation of the objective value to the following

$$\mathbb{E}\left[\sum_{R \in \mathcal{R}} \left\| \sum_{C_i \in \mathcal{C}} \frac{\mathbf{\Psi}_{a_{C_i}}}{l_{C_i}} \cdot \mathbf{E}_0(C_i, R) + \frac{\mathbf{\Psi}_{b_{C_i}}}{l_{C_i}} \cdot \mathbf{P}_{\pi/2} \cdot \mathbf{E}_0(C_i, R) \right\|^2\right]. \tag{11}$$

Denoting by $\mathbf{\Psi'}$ the $2m$-dimensional vector with the first m entries equal to $(\frac{\mathbf{\Psi}_{a_{C_i}}}{l_{C_i}} : i = 1, 2, \ldots, m)$ and the last m entries equal to $(\frac{\mathbf{\Psi}_{b_{C_i}}}{l_{C_i}} : i = 1, 2, \ldots, m)$, the above value in matrix form becomes

$$\mathbb{E}[\mathbf{Z} \bullet (\mathbf{\Psi'} \cdot \mathbf{\Psi'}^T)]. \tag{12}$$

Observe now that $\mathbf{\Psi'}$ is not multivariate normal and also, $\mathbf{\Psi'} \cdot \mathbf{\Psi'}^T$ is no longer an unbiased estimator for $\mathbf{Y_{opt}}$. However, since $\mathbf{\Psi}$ satisfies constraints $\mathbf{W}_{C_i} \bullet \mathbf{Y} = 1, i = 1, 2, \ldots, m$ on expectation (in particular, from probability theory for multivariate distribution we have $\mathbb{E}[\mathbf{W}_{C_i} \bullet \mathbf{\Psi}] = 1$, for every $i = 1, 2, \ldots, m$), implying that most of the l_{C_i} will be close to 1, we can hope to get to get a very good estimate of the optimal value $\mathbf{Z} \bullet \mathbf{Y_{opt}}$ by drawing enough samples from the multivariate distribution $\mathcal{N}(\mathbf{0}, \mathbf{Y_{opt}})$ and selecting the one that gives the best estimate. We note that sampling from $\mathcal{N}(\mathbf{0}, \mathbf{Y_{opt}})$ can be done efficiently in polynomial time in the size of the

[1] We can always ensure this by adding further linear constraints to our PSD program (9). This does not change the fact that an optimal solution can be computed efficiently within any required precision in polynomial time. Nevertheless, we chose not include symmetry constraints to facilitate exposition.

Input : $\mathbf{Z}, \mathbf{W}_{C_i}, i = 1, 2, \ldots, m$
Output: $\mathbf{\Phi}$
begin
  Find an optimal solution $\mathbf{Y_{opt}}$ to (9) ;
  Draw K samples $\mathbf{\Psi^{(1)}}, \mathbf{\Psi^{(2)}}, \ldots, \mathbf{\Psi^{(K)}}$ from $\mathcal{N}(\mathbf{0}, \mathbf{Y_{opt}})$;
  Compute $\mathbf{\Psi'^{(1)}}, \mathbf{\Psi'^{(2)}}, \ldots, \mathbf{\Psi'^{(K)}}$ as described in equation (12);
  let $k_{\max} = \arg\max_{k=1,2,\ldots,K} \left\{ \mathbf{Z} \bullet (\mathbf{\Psi^{(k)}}' \cdot \mathbf{\Psi^{(k)}}'^{T}) \right\}$;
  for $i = 1 : m$ **do**
    let ϕ_{C_i} be such that $\cos(\phi_{C_i}) = \mathbf{\Psi^{(k_{\max})}}'_{a_{C_i}}$ and $\sin(\phi_{C_i}) = \mathbf{\Psi^{(k_{\max})}}'_{b_{C_i}}$;
    $\mathbf{\Phi} = \mathbf{\Phi}^{-i}|\phi_{C_i}$;
  end
  return $\mathbf{\Phi}$;
end

Algorithm 1: ApproxPowerShift Algorithm

multivariate random variable, namely $2m$. Therefore, we get the ApprxPowerShift Algorithm (see Algorithm 1).

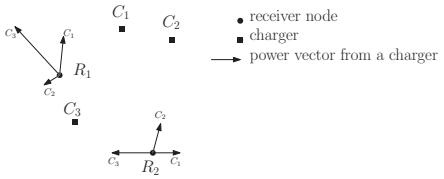
In particular, ApprxPowerShift uses standard semidefinite programming tools to compute an optimal solution $\mathbf{Y_{opt}}$ to the PSD program (9) and then draws K independent samples $\mathbf{\Psi^{(1)}}, \mathbf{\Psi^{(2)}}, \ldots, \mathbf{\Psi^{(K)}}$ from the multivariate distribution $\mathcal{N}(\mathbf{0}, \mathbf{Y_{opt}})$. For each such sample, it computes $\mathbf{\Psi^{(1)}}', \mathbf{\Psi^{(2)}}', \ldots, \mathbf{\Psi^{(K)}}'$ as described above and then selects the one (say $k_{\max}$) that maximizes $\max_{k=1,2,\ldots,K} \left\{ \mathbf{Z} \bullet (\mathbf{\Psi^{(k)}}' \cdot \mathbf{\Psi^{(k)}}'^{T}) \right\}$. Finally, the algorithm computes the phase shift for each charger by finding for each charger $C_i, i = 1, 2, \ldots, m$ the *unique* phase shift satisfying $\cos(\phi_{C_i}) = \mathbf{\Psi^{(k_{\max})}}'_{a_{C_i}}$ and $\sin(\phi_{C_i}) = \mathbf{\Psi^{(k_{\max})}}'_{b_{C_i}}$.

VII. DASA: A DISTRIBUTED ALGORITHM

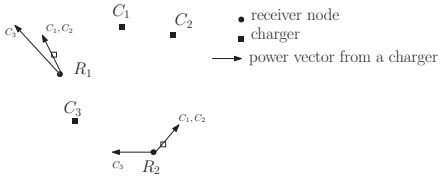
Here we describe a distributed algorithm, which we call Distributed phAse Shift Algorithm (DASA Algorithm 2), which can be used to find an approximation of an optimal solution to the `PowerShift` problem. We first need the following notation: for any phase shift configuration $\mathbf{\Phi}$, for any $i \in \{1, 2, \ldots, m\}$ and any $\phi \in [0, 2\pi)$, we denote by $\mathbf{\Phi}^{-i}|\phi$ the phase shift configuration that is equal to $\mathbf{\Phi}[j]$, for all $j \neq i, j \in \{1, 2, \ldots, m\}$ and $\mathbf{\Phi}[i] = \phi$, i.e. $\mathbf{\Phi}^{-i}|\phi$ comes from $\mathbf{\Phi}$ by replacing $\mathbf{\Phi}[i]$ with the phase shift ϕ.

To facilitate exposition, we assume that DASA works in rounds (however, this is not restrictive and DASA can work also in an asynchronous, distributed system, ex: in every specific or random period of time, a charger communicates with a set of nodes and calibrates its initial phase). For $t = 0, 1, \ldots$, we denote by $\mathbf{\Phi}_t$ the phase shift configuration of the chargers at round t. Initially, we assign the phase shift 0 to every charger $C \in \mathcal{C}$, i.e. we set $\mathbf{\Phi}_0[i] = 0$, for all $i = 1, 2, \ldots, m$. Notice that, given $\mathbf{\Phi}$, we can compute in polynomial time the cumulative power from the set of chargers $\mathcal{C}$ to the set of nodes $\mathcal{R}$, namely $P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R})$, using equations (1) to (4) in the definition of the charging model.

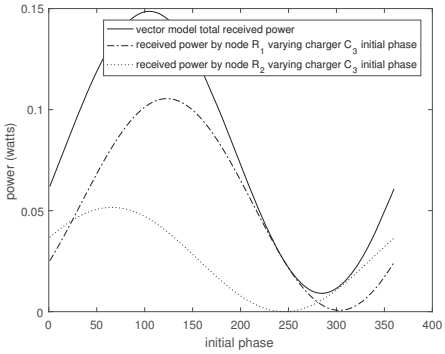
In every round, a charger C_i is randomly chosen such that it can potentially increase the total cumulative power by



(a) Power vectors from the corresponding chargers are pictured at the nodes' location.



(b) Power vectors from the corresponding chargers are pictured at the nodes' location. Squared vector equals to the vectorial sum of C_1 *and* C_2 chargers.



(c) Power vectors from the respective chargers are pictured at the nodes' location.

Figure 1: A WPT network in which we depict the power vectors at the receivers' location.

unilaterally adjusting the phase shift of C_i. In particular, this happens if for some charger C_i, we have

$$\max_{\phi \in [0,2\pi)} \{P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R})\} > P_{\mathbf{\Phi}}(\mathcal{C}, \mathcal{R}). \tag{13}$$

If such a charger exists, say C_i, then we adjust its phase shift to the value ϕ^* that maximizes $P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R})$, i.e. we set $\mathbf{\Phi}_{t+1} = \mathbf{\Phi}_t^{-i}|\phi^*$, and we proceed to the next round. Otherwise, if there is no charger that can increase the cumulative power by unilaterally adjusting its phase, then the algorithm terminates. This approach works in rounds to facilitate the implementation process, since in a distributed setting a charger can individually reconfigure its parameter after a communication phase.

We now show how the computation needed to find the maximum in equation (13) can be done efficiently and locally for each charger C_i by gathering all required information from other nodes in the system. In particular, the information that a charger needs from every node $R \in \mathcal{R}$ is the vector of the energy field created at R, namely $\mathbf{E}_{\Phi}(\mathcal{C}, R)$, where Φ

is the current phase shift configuration (note that the node does not have to know $\mathbf{\Phi}$ to find $\mathbf{E}_\Phi(\mathcal{C}, R)$). From this and knowledge of the distance $\text{dist}(C_i, R)$, the charger can deduce $\mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R)$, namely the vector of the electric field created to R by all other chargers except C_i. Therefore, we can rewrite $P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R})$ in the maximum from equation (13) as

$$\sum_{R\in\mathcal{R}} \gamma \|\mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R) + \mathbf{E}_\phi(C_i, R)\|^2 \tag{14}$$

or equivalently

$$\sum_{R\in\mathcal{R}} \gamma\Big(\|\mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R)\|^2 + \|\mathbf{E}_\phi(C_i, R)\|^2 + 2\mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R)^T \mathbf{E}_\phi(C_i, R)\Big). \tag{15}$$

Since a charger can unilaterally change its phase shift, when C_i computes the above summation we treat $\mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R)$ as a constant vector. Additionally, since $\|\mathbf{E}_\phi(C_i, R)\|^2$ is independent of the phase shift ϕ for charger C_i, the only term containing variables in equation (15) is $\sum_{R\in\mathcal{R}} 2\gamma \mathbf{E}_\Phi(\mathcal{C}\backslash C_i, R)^T \mathbf{E}_\phi(C_i, R)$. But by equation (6), the latter summation is just a linear combination of $\cos(\phi)$ and $\sin(\phi)$, i.e. it is of the form $A\cos(\phi) + B\sin(\phi)$, for some constants $A, B \in \mathbb{R}$. From elementary trigonometry, this can be written as $\sqrt{A^2+B^2}\sin(\phi + x)$, where x is the unique phase shift such that $\cos(x) = \frac{A}{\sqrt{A^2+B^2}}$ and $\sin(x) = \frac{B}{\sqrt{A^2+B^2}}$. Therefore, the phase shift ϕ that maximizes $A\cos(\phi) + B\sin(\phi)$ and consequently also $P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R})$ is $\phi^* = \frac{\pi}{2} - x$. We have thus proved the following:

Lemma 1. Assume that each node $R \in \mathcal{R}$ can determine $\mathbf{E}_\Phi(\mathcal{C}, R)$ upon request. Then every charger $C_i, i = 1, 2, \ldots, m$, can determine in polynomial time the optimal value $\phi^* \in [0, 2\pi)$ that maximizes $P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R})$. In particular, it can determine in polynomial time whether (13) holds.

An example of how the above Lemma is applied is shown in Figure 1a. In particular, in that figure we can see examples of the electric field vectors which are formed at each node. Suppose that we would like to find the best phase shift for charger C_3 given fixed phase shifts for all other chargers. In particular, charger C_3 knows the electric field vectors at every node, and thus is able to find the vectors shown in Figure 1b. Figure 1c depicts the received power at each of the two nodes (dotted lines) as well as the cumulative received power (solid line) for different phase shifts of C_3. As we showed in the above Lemma, the solid line corresponds to a sinusoidal function since it is a sum of sines and cosines of the same frequency. For this particular toy example the best phase for C_3 is 106.

As an additional feature, in the DASA algorithm we employ the *communication radius* parameter ρ. Using this parameter we can study our distributed algorithm for different levels of network knowledge from chargers, in the sense that chargers can exchange messages and information only with other nodes that are within distance ρ. In particular, the information that a charger can now get consists only of the electric field vectors $\mathbf{E}_\Phi(\mathcal{C}, R)$ from nodes R such that $\text{dist}(C, R) \leq \rho$. Notice then that the summation in equation (14) computing cumulative power is restricted to a subset of $\mathcal{R}$. Nevertheless, the larger ρ is, the smaller the effect of a phase shift for C_i on those distant nodes will be. In return, the larger ρ is, the closer the (approximate) optimal phase shift computed by C_i with partial information to the global optimal phase shift for C_i using global information (i.e. with $\rho \to \infty$) will be. This intuition is also supported by our experimental evaluation.

Input : dist, $\mathcal{R}$, $\mathcal{C}$, ρ
Output: $\mathbf{\Phi}$
begin
  $\mathbf{\Phi}[i] = 0$ for $i = 1, 2, \ldots, m$ // initialization phase;
  while $\exists C_i \in \mathcal{C}$: *(13) holds* **do**
    choose randomly $C_j \in \mathcal{C}$;
    let $\mathcal{R}_{C_j,\rho} = \{R \in \mathcal{R} : \text{dist}(C_j, R) \leq \rho\}$;
    $\phi^* = \arg\max_{\phi\in[0,2\pi)} P_{\mathbf{\Phi}^{-i}|\phi}(\mathcal{C}, \mathcal{R}_{C_j,\rho})$;
    $\mathbf{\Phi} = \mathbf{\Phi}^{-i}|\phi^*$;
  end
  return $\mathbf{\Phi}$;
end

Algorithm 2: Distributed phAse Sift Algorithm (DASA)

Finite set of available phase shifts. For comparison purposes, we also propose the Discrete Phase Algorithm, that uses DASA in order to determine a phase shift configuration $\mathbf{\Phi}'$, which achieves large cumulative power and all assigned phases belong to a set of predetermined phase shifts $\mathcal{S} \stackrel{\text{def}}{=} \{0, \frac{\pi}{2}, \pi, \frac{2\pi}{3}\}$ (clearly this can be generalized to any set of predetermined phase shifts). We improve upon the naive idea of optimizing each charger separately over $\mathcal{S}$ as follows: We define a set F where we include (one at a time) chargers with fixed phase shifts. In particular, once a charger C enters F, it will have a phase shift in $\mathcal{S}$ which will not be altered until termination of the algorithm. Consequently, DASA treats the phases of chargers in F as constant values; the latter means that they are not assigned to 0 in the initialization phase of DASA, and neither can chargers in F be the chargers considered in the while-loop condition or the ones selected randomly inside that loop. To determine which charger to include next into F, we run DASA (regarding phase shifts of chargers already in F as constants) and then select the charger $C \in \mathcal{C}\backslash F$ whose phase shift is the closest to the phases in $\mathcal{S}$. Intuitively, "rounding" the phase shift of C to one of the values in $\mathcal{S}$ is expected to have minimal effect on the cumulative power. This intuition is also supported by our experimental evaluation.

VIII. Evaluation

We use Matlab2016a for the simulations process that evaluates the performance of our algorithms in a $10m^2$ square field. We assume that each charger transmits at 915 MHz frequency, meaning that the wavelength equals to 32cm. Charger delivers energy via an isotropic antenna of 2 Watts power and 2 dbi gain, while on the other hand, the gain of the receiver node is 1 dbi.

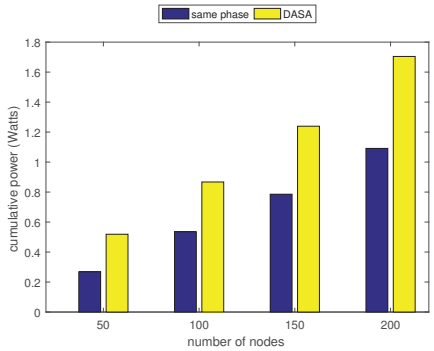


Figure 2: Benchmark static/fixed phase and DASA over different number of receivers in a system.

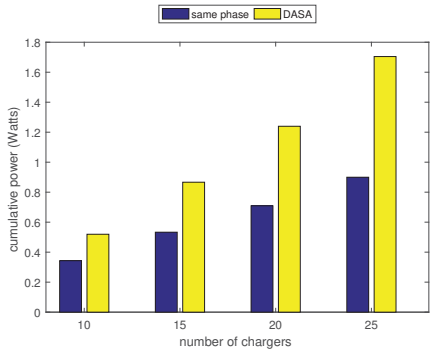


Figure 3: Benchmark static/fixed phase and DASA over different number of chargers in a system.

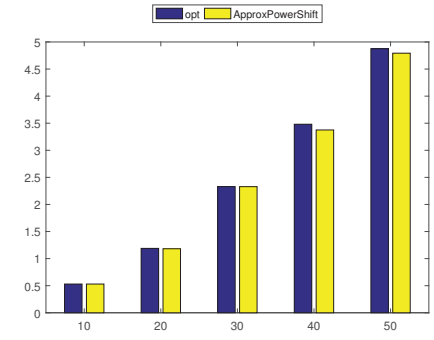


Figure 4: The power performance between the upper bound provided by Corollary 1 (opt) and ApproxPowerShift algorithm varying the number of chargers.

In the analysis below, we aim to present the performance of our designed algorithms regarding the cumulative received power by the nodes as the main and critical benchmark metric. We start by presenting in Figures 2,3 the motivation of designing such algorithms that follow a more accurate model and take into advantage the intuition that raises from it. As we see, DASA outperforms the results of the *same phases* setting for both cases with different number of chargers and nodes. In particular for Figure 2 we deploy uniformly at random 15 chargers and 50 receivers in the $10m^2$ square field. For the next bar-pair, we similarly deploy 15 chargers and 100 receivers and so forth for the next two bar-pairs. On the contrary, in Figure 3 we have 100 receivers and we deploy different number of chargers. For statistical analysis, the above experiment was performed 100 times showing a good concentration around the mean. Thus, we conclude that the improvement is substantial.

In a network of 100 receivers and a number of chargers that ranges from 10 to 50, we conducted a bench-marking simulation for Algorithm 1 performance against optimal high bound. The simulation ran for 100 executions and in Figure 4 we present the mean values as we resulted high concentration. Thus, for a smaller number than 30 chargers, Algorithm 1 achieves the optimal, while for over 40 chargers a near optimal performance. Note that *opt* refers to the upper bound provided by Corollary 1. In Figure 5 we show the cumulative received power performance by 100 nodes for different communication radii of 7 chargers for a 90 rounds period of time. Notice that in most of our experiments DASA achieves close to optimal cumulative power when the communication radius is large (global communication radius), but as the communication radius decreases we get slightly smaller values. We can see that the cumulative power with the global communication radii never decreases over time like the others do. This is because every charger has global knowledge of the power exchange in the system. Indeed, when a charger has limited communication radius, then its phase shift choice will be based on information from nodes in the neighborhood, and does not take into account nodes further away (where this choice may cause cancellation effects). The simulated data presented here are averages over 100 executions which corresponds to 100 different random chargers sequences.

For the simulation corresponding to Figure 6, we configured our algorithm so that the phase shift can take values from a predetermined set of values evenly distributed in $[0, 2\pi)$. As expected, the performance of our algorithm improves as the number of different phase shifts that a charger can choose from increases. As we can see, the distributed algorithm has a good approximation while the heuristic is slightly better. Here we present averaged values from 100 executions, as we resulted high concentration. Finally, in contrast to this simulation, Figure 5 presents fluctuations, as the cumulative power may decrease during the running time. This is consequence of the bounded communication radius and it does not occur in the ideal case where the communication radius is infinite.

IX. Conclusion and Future Work

In this paper we apply vector model with respect to the phase sifting technique for the first time and study *PowerShift Problem* in WPT networks. We present upper performance

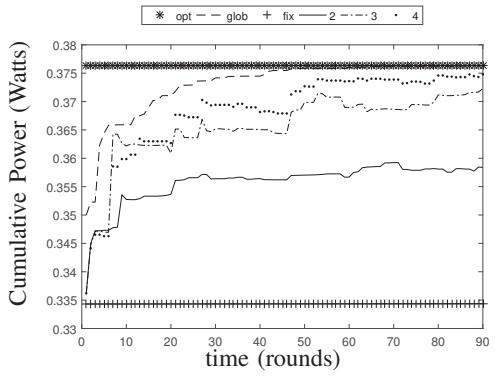


Figure 5: DASA algorithm performance over difference communication radii. High bound and fixed phases performances are also included.

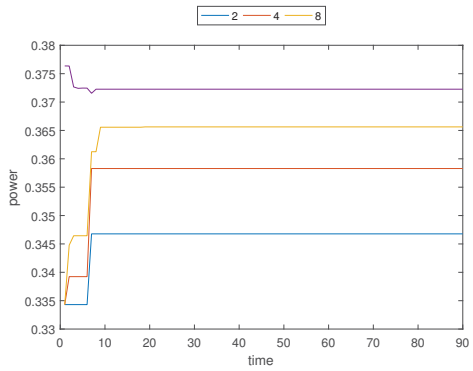


Figure 6: DAH (8 potential phase shifts) and DASA (2, 4, 8 potential phase shifts) performance over 90 rounds time.
DAH 2 4, 8

bounds, heuristics and an extended numerical simulation, comparing different approaches and settings.

In future we opt to further investigate vector model, combined to phase sifting feature to deal with a) more sophisticated power management approaches in networks that make use of rf signals and b) its potential regarding the EMR concept. This work strongly enables radiation awareness in a variety of problems in WPT concept that skip this aspect. Finally, we also wish to add more restrictions and study trade-offs such as efficient charging alongside EMR safety even though it is discouraging cause of the complexity.

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